答案与提示

习题 8-1

1. $\sqrt{34}$，$\sqrt{41}$，5.
2. $(0,1,-2)$.

习题 8-2

1. $D=(1,0,1)$.
2. $2\sqrt{19}$.
3. (1) $\{3,0,6\}$. (2) $\{-1,4,0\}$. (3) $\{-4,10,-3\}$.
4. 向量$\overrightarrow{AB}$的模为 2；方向余弦 $\cos\alpha=-\frac{1}{2}$，$\cos\beta=\frac{1}{2}$，$\cos\gamma=-\frac{\sqrt{2}}{2}$；方向角 $\alpha=\frac{2\pi}{3}$，$\beta=\frac{\pi}{3}$，$\gamma=\frac{3\pi}{4}$.
5. $\{2,-4,6\}$.
6. (1) -1. (2) 4. (3) 1.
7. 略.
8. (1) -4. (2) $-7\boldsymbol{i}-9\boldsymbol{j}-4\boldsymbol{k}$. (3) $-\frac{4}{\sqrt{18}}$. (4) $-\frac{7}{\sqrt{146}}\boldsymbol{i}-\frac{9}{\sqrt{146}}\boldsymbol{j}-\frac{9}{\sqrt{146}}\boldsymbol{k}$. (5) $\boldsymbol{c}=\{10,-6,-4\}$. (6) 101.
9. 略.
10. $\sqrt{14}$.
11. 4.

习题 8-3

1. $x+y-3z-4=0$.
2. $14x+9y-z-15=0$.
3. $2y+z+3=0$.
4. (1) $y-3z=0$. (2) $9y-z-2=0$. (3) $47x+13y+z=0$.
5. $x+2y+2z-2=0$.
6. $6x+3y-10z-7=0$.
7. $\frac{1}{3},\frac{2}{3},\frac{2}{3}$.

习题 8-4

1. $\frac{x-1}{2}=\frac{y-1}{-3}=\frac{z}{1}$.

2. $\frac{x}{-2}=\frac{y-2}{3}=\frac{z-4}{1}$.

3. $\frac{x}{1}=\frac{y}{2}=\frac{z+1}{1}$.

4. $-6x+3y+z=0$.

5. 0.

6. (1) $2x-z-5=0$. (2) $3x+3y-z+2=0$.

7. 0.

8. (1) 平行. (2) 垂直. (3) 直线在平面上.

9. $\frac{x-1}{-1}=\frac{y-2}{-1}=\frac{z+1}{1}$.

习题 8-5

1. 略

2. $4x+4y+10z-63=0$.

3. 以点$(1,-2,-1)$为球心,半径为$\sqrt{6}$的球面.

4. $y^2+z^2=5x$.

5. $x^2+y^2+z^2=9$.

6. $z=\frac{2}{3}x^2+\frac{1}{3}y^2$.

7. (1) 旋转曲面. Oxy 平面上的圆 $x^2+y^2=1$ 绕 x 或 y 轴旋转一周,或 Oyz 平面上的圆 $y^2+z^2=1$ 绕 y 或 z 轴旋转一周,或 Ozx 平面上的圆 $x^2+z^2=1$ 绕 x 或 z 旋转一周.

   (2) 非旋转曲面.

   (3) 旋转曲面. Oxy 平面上的双曲线 $x^2-\frac{y^2}{4}=1$ 绕 y 旋转一周,或 Oyz 平面上的双曲线 $z^2-\frac{y^2}{4}=1$ 绕 y 旋转一周.

   (4) 旋转曲面. Oxy 平面上的双曲线 $x^2-y^2=1$ 绕 x 轴旋转一周,或 Ozx 平面上的双曲线 $x^2-z^2=1$ 绕 x 轴旋转一周.

习题 8-6

1. $4x^2-9y^2=36$.

2. $\begin{cases} y^2=2x-9, \\ z=0. \end{cases}$ 原曲线是位于平面 $z=3$ 上的抛物线.

3. $z=0, x^2+y^2=x+y$. $x=0, 2y^2+2yz+z^2-4y-3z+2=0$.
   $y=0, 2x^2+2xz+z^2-4x-3z+2=0$.

4. $z=0, (x-1)^2+y^2\leqslant 1; x=0, \left(\frac{z^2}{2}-1\right)^2+y^2\leqslant 1, z\geqslant 0; y=0, 0\leqslant x\leqslant 2$.

5. $\begin{cases} x^2+2y^2-2y=0, \\ z=0. \end{cases}$

复习题八

一、选择题

1—5：BAABD.　6—10：BCBBC.　11—15：DCCAD.

二、综合练习 A

1. $\frac{6}{7}$.

2. $-\frac{3}{2}$.

3. 略.

4. $\frac{\pi}{3}$.

5. $x-y+z=0$.

6. $D=3$.

7. $21x+14z-3=0$.

8. $\frac{x+1}{3}=\frac{y-2}{-1}=\frac{z-1}{1}$.

三、综合练习 B

1. 略.

2. $\pm\left\{\frac{2}{\sqrt{16}},\frac{1}{\sqrt{16}},\frac{1}{\sqrt{16}}\right\}$.

3. 略.

4. $y-z-1=0$.

5. $\begin{cases}17x+31y-37z-117=0,\\4x-y+z-1=0.\end{cases}$

6. $z=2$ 或 $132x+176y-21z-442=0$.

7. L：$\frac{x}{1}=\frac{y+1}{0}=\frac{z-1}{2}$，$d=\sqrt{5}$.

8. $\frac{x-2}{-1}=\frac{y-1}{-2}=\frac{z+1}{1}$.

9. $x^2+y^2+z^2-4yz-4zx-4xy=0$. 这是半顶角为 $\frac{\pi}{4}$，以 OM 为对称轴的正圆锥面.

习题 9-1

1. (1) $\{(x,y)\mid y^2-2x+1>0\}$.　(2) $\{(x,y)\mid |x|+|y|<1\}$.

   (3) $\{(x,y)\mid |y|\leqslant|x|,x\neq0\}$.　(4) $\{(x,y)\mid x^2\leqslant y\leqslant4\}$.

   (5) $\{(x,y,z)\mid r<x^2+y^2+z^2\leqslant R\}$.

2. $f(2,1)=0$，$f(3,-1)=\frac{5}{7}$.

3. $f\left(1,\frac{y}{x}\right)=f(x,y)$，$x\neq0$.

4. $f[g(1,2)]=g[f(1),h(2)]=6$.

5. 略.

6. (1) 6. (2) 1. (3) 2. (4) 0. (5) 2. (6) e^k. (7) e.

7. 略.

8. 略.

9. 在圆周$(x-1)^2+y^2=1$上.

10. 略.

11. 略

习题 9-2

1. (1) $z_x=y-\frac{1}{y}$, $z_y=x+\frac{x}{y^2}$.

(2) $z_x=y\ln(x^2+y^2)+\frac{2x^2y}{x^2+y^2}$, $z_y=x\ln(x^2+y^2)+\frac{2xy^2}{x^2+y^2}$.

(3) $z_x=y^2(1+xy)^{y-1}$, $z_y=(1+xy)^y\left[\ln(1+xy)+\frac{xy}{1+xy}\right]$.

(4) $\frac{\partial z}{\partial x}=e^{-xy}[\cos(x-y)-y\sin(x-y)]$, $\frac{\partial z}{\partial y}=-e^{-xy}[\cos(x-y)+x\sin(x-y)]$.

(5) $\frac{\partial z}{\partial x}=-\frac{y}{x^2+y^2}$, $\frac{\partial z}{\partial y}=\frac{x}{x^2+y^2}$.

(6) $\frac{\partial s}{\partial u}=\frac{1}{v}-\frac{v}{u^2}$, $\frac{\partial s}{\partial v}=\frac{1}{u}-\frac{u}{v^2}$.

(7) $\frac{\partial z}{\partial x}=ye^{-x^2y^2}$, $\frac{\partial z}{\partial y}=xe^{-x^2y^2}$.

(8) $\frac{\partial z}{\partial x}=\frac{1}{2x\sqrt{\ln(xy)}}$, $\frac{\partial z}{\partial y}=\frac{1}{2y\sqrt{\ln(xy)}}$.

(9) $\frac{\partial z}{\partial x}=ye^{\sin(xy)}\cos(xy)$, $\frac{\partial z}{\partial y}=xe^{\sin(xy)}\cos(xy)$.

(10) $\frac{\partial z}{\partial x}=\frac{2}{x}\ln\frac{x}{y}$, $\frac{\partial z}{\partial y}=-\frac{2}{y}\ln\frac{x}{y}$.

(11) $\frac{\partial u}{\partial x}=\frac{y}{z}x^{\frac{y}{z}-1}$, $\frac{\partial u}{\partial y}=\frac{1}{z}x^{\frac{y}{z}}\ln x$, $\frac{\partial u}{\partial z}=-\frac{y}{z^2}x^{\frac{y}{z}}\ln x$.

(12) $\frac{\partial u}{\partial x}=\frac{z(x-y)^{z-1}}{1+(x-y)^{2z}}$, $\frac{\partial u}{\partial y}=-\frac{z(x-y)^{z-1}}{1+(x-y)^{2z}}$, $\frac{\partial u}{\partial z}=\frac{(x-y)^z\ln|x-y|}{1+(x-y)^{2z}}$.

2. (1) $f_x(2,3)=36$. (2) $f_x(3,4)=\frac{1}{5}$, $f_y(3,4)=\frac{1}{10}$.

3. $f_x(x,1)=\frac{df(x,1)}{dx}=1$.

4. 略.

5. (1) $z_{xx}=12x^2-8y^2$, $z_{xy}=-16xy$, $z_{yy}=12y^2-8x^2$.

(2) $z_{xx}=24x+6y$, $z_{xy}=6x-6y$, $z_{yy}=-6x$.

(3) $z_{xx}=\frac{x+2y}{(x+y)^2}$, $z_{xy}=\frac{y}{(x+y)^2}$, $z_{yy}=-\frac{x}{(x+y)^2}$.

(4) $z_{xx}=(2-y)\cos(x+y)-x\sin(x+y)$, $z_{yy}=-(2+x)\sin(x+y)-y\cos(x+y)$,
$z_{xy}=(1-y)\cos(x+y)-(1+x)\sin(x+y)=z_{yx}$.

6－8. 略.

习题 9-3

1. (1) $\frac{\sqrt{xy}}{2xy^2}(y\mathrm{d}x-x\mathrm{d}y)$.　　(2) $\sec^2(x^2y)(2xy\mathrm{d}x+x^2\mathrm{d}y)$.

(3) $\frac{ab}{\sqrt{(ax+by)(ax-by)^3}}(-y\mathrm{d}x+x\mathrm{d}y)$.　　(4) $y^2x^{y^2-1}dx+2yx^{y^2}(\ln x)\mathrm{d}y$.

(5) $2\arctan\frac{x}{y}\cdot\frac{1}{x^2+y^2}(y\mathrm{d}x-x\mathrm{d}y)$.

2. 略.

3. $\mathrm{d}x-\mathrm{d}y$.

4. $\frac{1}{3}\mathrm{d}x+\frac{2}{3}\mathrm{d}y$.

5. $\mathrm{d}z=-0.20$.

6. (1) 108.9.　(2) 2.003 3.

7. 5 cm.

习题 9-4

1. (1) $\frac{\mathrm{d}z}{\mathrm{d}x}=\frac{y(1+x)}{1+x^2y^2}$.

(2) $\frac{\mathrm{d}z}{\mathrm{d}t}=\frac{1}{y}\cdot c+\left(-\frac{x}{y^2}\cdot\frac{1}{t}\right)=\frac{c}{\ln t}-\frac{c}{(\ln t)^2}$.

(3) $\frac{\partial z}{\partial x}=2x^3\sin 2y,\frac{\partial z}{\partial y}=x^4\cos 2y$.

(4) $\frac{\partial z}{\partial x}=\mathrm{e}^{xy}[y\sin(x+y)+\cos(x+y)],\frac{\partial z}{\partial y}=\mathrm{e}^{xy}[x\sin(x+y)+\cos(x+y)]$.

(5) $\frac{\partial z}{\partial x}=3u^2y^x\ln y=3u^3\ln y$, $\frac{\partial z}{\partial y}=3u^2xy^{x-1}=\frac{3u^3x}{y}$.

(6) $\frac{\partial z}{\partial x}=\frac{x}{\sqrt{x^2+2y^2}}\ln\frac{x}{y}+\frac{\sqrt{x^2+2y^2}}{x},\frac{\partial z}{\partial y}=\frac{2y}{\sqrt{x^2+2y^2}}\ln\frac{x}{y}-\frac{\sqrt{x^2+2y^2}}{y}$.

2－4. 略.

5. (1) $\frac{\partial w}{\partial x}=2xf'_1+y\mathrm{e}^{xy}f'_2$, $\frac{\partial w}{\partial y}=-2yf'_1+x\mathrm{e}^{xy}f'_2$.

(2) $\frac{\partial z}{\partial x}=f'_1+\frac{1}{y}f'_2+2xy\varphi',\frac{\partial z}{\partial y}=-\frac{x}{y^2}f'_2+\varphi-2y^2\varphi'$.

*6. (1) $z_{xx}=y^2f''_{11}$, $z_{xy}=f'_1+y(xf''_{11}+f''_{12})$, $z_{yy}=x^2f''_{11}+2xf''_{12}+f''_{22}$.

(2) $z_{xx}=2f-\frac{2y}{x}f'+\frac{y^2}{x^2}f''$, $z_{xy}=f'-\frac{y}{x}f''$, $z_{yy}=f''$.

7. 略.

8. $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{\mathrm{e}^x-y^2}{\cos y-2xy}$.

9. $\frac{\partial z}{\partial x}=-\frac{1-y}{3z^2-2}=\frac{y-1}{3z^2-2}$, $\frac{\partial z}{\partial y}=-\frac{2y-x}{3z^2-2}=\frac{x-2y}{3z^2-2}$.

10. $\frac{\partial z}{\partial x}=-\frac{x^2+2yz}{z^2+2xy}$, $\frac{\partial z}{\partial y}=-\frac{y^2+2xz}{z^2+2xy}$.

11. $\frac{\partial z}{\partial x}=\frac{yz}{e^x-xy}$, $\frac{\partial z}{\partial y}=\frac{xz}{e^x-xy}$.

12. $dz=\frac{dx-ze^{yz}dy}{2z+ye^{yz}}$.

13. $\frac{1}{5}$.

14. (1) $\left.\frac{dy}{dx}\right|_{(1,1,\sqrt{2})}=0$, $\left.\frac{dz}{dx}\right|_{(1,1,\sqrt{2})}=-\frac{\sqrt{2}}{2}$.

(2) $\frac{\partial u}{\partial x}=\frac{\sin v}{e^u(\sin v-\cos v)+1}$, $\frac{\partial u}{\partial y}=\frac{-\cos v}{e^u(\sin v-\cos v)+1}$.

$\frac{\partial v}{\partial x}=\frac{\cos v-e^u}{u[e^u(\sin v-\cos v)+1]}$, $\frac{\partial v}{\partial y}=\frac{\sin v+e^u}{u[e^u(\sin v-\cos v)+1]}$.

15. 压力以 0.042 kPa/s 的速率减少.

16. 略.

习题 9-5

1. $\frac{3}{4}\pi$.

2. $\left\{\frac{3}{4},\frac{1}{4},-1\right\}$.

3. 切线：$\frac{x-\frac{\pi}{2}+1}{1}=\frac{y-1}{1}=\frac{z-2\sqrt{2}}{\sqrt{2}}$；法平面：$x+y+\sqrt{2}z=\frac{\pi}{2}+4$.

4. 切线：$\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-3}{3}$；法平面：$(x-1)+(y-1)+3(z-3)=0$.

5. 切平面：$-2(x-1)-4(y-2)+z-5=0$；法线：$\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-5}{1}$.

6. 切平面方程：$x-3z=5$；

法线方程：$\frac{x-2}{1}=\frac{y}{0}=\frac{z+1}{-3}$ 或 $\begin{cases}x-2=-\frac{1}{3}(z+1),\\ y=0.\end{cases}$

7. 切平面方程：$2x+y-3z+6=0$ 和 $2x+y-3z-6=0$.

8. 切平面方程：$2x-y-2z=\frac{5}{8}$.

习题 9-6

1. (1) 极大点$(2,-2)$，极大值 8.

(2) 极小点$\left(\frac{1}{2},-1\right)$，极小值$-\frac{e}{2}$.

(3) 极大点$(3,2)$，极大值 36.

(4) 极小点$(0,2)$，极小值-2；极大点$(0,0)$，极大值 2.

2. 最大值 $z(0,3)=z(3,0)=6$，最小值 $z(1,1)=-1$.

3. 所求点为$\left(\frac{8}{5},\frac{3}{5}\right)$,最短距离为$\frac{\sqrt{13}}{13}$.

4. 长、宽、高都为$\frac{2a}{\sqrt{3}}$.

5. 当矩形的边长为$\frac{2}{3}p$及$\frac{p}{3}$时，绕短边旋转而构成的圆柱体体积最大.

6. 正面长为$2\sqrt{10}$ m、侧面长为$3\sqrt{10}$ m时，所用材料费最少.

7. 当长为$\frac{4}{17}\sqrt{\frac{5a}{k}}$，宽(深)为$\frac{1}{6}\sqrt{\frac{5a}{k}}$时,可使容积最大.

习题 9-7

1. $\frac{1}{2}(5+3\sqrt{2})$.

2. $\frac{98}{13}$.

3. 1.

4. (8,5,6).

5. 沿方向(2,−4,1)的方向导数最大,最大值为$\sqrt{21}$.

6. $\frac{1}{ab}\sqrt{2(a^2+b^2)}$. 提示:可将椭圆看作空间坐标系中的椭圆柱面$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$与$Oxy$平面的交线，其上任一点处的法线向量为$\left(\frac{2x}{a^2},\frac{2y}{b^2}\right)$,因此在点$\left(\frac{a}{\sqrt{2}},\frac{b}{\sqrt{2}}\right)$处的法线向量为$\left(\frac{a}{\sqrt{2}},\frac{b}{\sqrt{2}}\right)$,内法线方向单位向量为$\left(-\frac{b}{\sqrt{a^2+b^2}},-\frac{a}{\sqrt{a^2+b^2}}\right)$,由此求得方向导数为$\frac{1}{ab}\sqrt{2(a^2+b^2)}$.

7. (1) $-\frac{40}{3\sqrt{3}}$. (2) 略.

8. 略.

复习题九

一、选择题

1—5：AAABD. 6—10：CDCCB. 11—14：BBDD.

二、综合练习 A

1. $\frac{\partial z}{\partial x}=\frac{\cos(z+y-x)^2}{\cos(z+y-x)^2-2}$.

2—3. 略.

4. 切平面方程 $2x+y-3z+6=0$ 和 $2x+y-3z-6=0$.

5. 最大值 8,最小值 0.

三、综合练习 B

1—2. 略.

3. $1,a+ab+ab^2+b^3$.

4. $\mathrm{d}z=\frac{z}{z-1}\left(\frac{1}{x}\mathrm{d}x+\frac{1}{y}\mathrm{d}y\right)$,$\frac{\partial^2 z}{\partial^2 x^2}=\frac{2y^2ze^z-2xy^3z-y^2z^2e^x}{(e^z-xy)^3}$.

5. (1,2).

6—7. 略.

习题 10-1

1. (1) 大于零. (2) 小于零. (3) 小于零. (4) 大于零.

2. (1) $I_1 \geqslant I_2$. (2) $I_1 \leqslant I_2$. (3) $I_1 \geqslant I_2$. (4) $I_1 \leqslant I_2$.

3. (1) $0 \leqslant I \leqslant \frac{\pi}{4}e^{\frac{1}{4}}$. (2) $2 \leqslant I \leqslant 8$. (3) $36\pi \leqslant I \leqslant 100\pi$.

4. (1) $I=2I_1$. (2) $I=4I_1$. (3) $I=I_1=0$.

*5. 正确的等式有(1)、(4)、(5)、(6).

习题 10-2

1. (1) $\frac{1}{2}(1-e^{-1})$. (2) $e-3e^{-1}$. (3) $\frac{1}{2}(e^2-2)e^2$. (4) $\frac{1}{15}$.

(5) $\frac{45}{8}$. (6) $\frac{2}{15}(4\sqrt{2}-1)$. (7) $\frac{\pi}{2}$.

2. (1) $(e^{b^2}-e^{a^2})\pi$. (2) $\frac{32}{9}R^3$. (3) $\frac{\pi}{2}(2\ln 2-1)$. (4) $\frac{1}{48}(4+3\pi)$.

(5) $\frac{1}{6}\pi a^3$. (6) $\frac{10\sqrt{2}}{9}$.

3. (1) $\int_0^2 dy\int_0^y f(x,y)dx$. (2) $\int_0^1 dx\int_0^{x^2} f(x,y)dy+\int_1^{\sqrt{2}} dx\int_0^{\sqrt{2-x^2}} f(x,y)dy$.

(3) $\int_0^1 dy\int_{e^y}^{e} f(x,y)dx$. (4) $\int_0^1 dy\int_y^{2-y} f(x,y)dx$. (5) $\int_0^1 dx\int_0^{1-x^2} 3x^2y^2dy$.

4. (1) $\frac{76}{3}$. (2) $2+\pi\ln 2$. (3) $\frac{a^2}{64}\pi^2$. (4) $\frac{20\sqrt{2}}{9}$.

5. (1) $\frac{e-2}{6e}$. (2) $\frac{1}{4}(2-\sqrt{2})$. (3) $\frac{1}{4}(e-1)$.

习题 10-3

1. (1) 2π. (2) $\frac{1}{9}(3\pi-4)a^3$. (3) 4π.

2. (1) $\sqrt{3}\pi$. (2) $\frac{2}{3}(5\sqrt{5}-1)\pi$.

△3. $\left(0,\frac{8\ln 2-3}{8\ln 2-4}\right)$. △4. $\left(\frac{7}{3},0\right)$. △5. $\frac{1}{3}ab^3,\frac{1}{3}a^3b$.

习题 10-4

1. (1) $\iint\limits_D d\sigma\int_{x^2+y^2}^{1} f(x,y,z)dz, D=\{(x,y)\mid x^2+y^2\leqslant 1\}$.

(2) $\iint\limits_D d\sigma\int_0^{x^2+y^2} f(x,y,z)dz, D=\{(x,y)\mid -1\leqslant x\leqslant 1, x^2\leqslant y\leqslant 1\}$.

(3) $\iint\limits_D \mathrm{d}\sigma\int_0^{\frac{\pi}{2}-x} f(x,y,z)\mathrm{d}z, D=\left\{(x,y)\mid 0\leqslant x\leqslant\frac{\pi}{2}, 0\leqslant y\leqslant\sqrt{x}\right\}$.

(4) $\iint\limits_D \mathrm{d}\sigma\int_{x^2+2y^2}^{2-x^2} f(x,y,z)\mathrm{d}z, D=\{(x,y)\mid x^2+y^2\leqslant 1\}$.

2. (1) 2. (2) $\frac{1}{2}\left(\ln 2-\frac{5}{8}\right)$. (3) $\frac{1}{2}\ln 2$. (4) $\frac{\pi}{8}R^4$.

(5) $\frac{208}{15}\pi$. (6) $\frac{16}{3}\pi$. (7) πR^4. (8) $\frac{\pi}{10}$.

3. $k\pi R^4$.

复习题十

一、选择题

1—5：BBBAD. 6—10：ADDBC. 11—15：CABDB.

二、综合练习 A

1. $\frac{2}{3}$.

2. $\frac{2}{5}$.

3. $\frac{41\pi}{2}$.

4. 略

5. $\frac{1}{2}\left(1-\frac{1}{\mathrm{e}}\right)$.

三、综合练习 B

1. $\frac{49}{32}\pi\mu a^4$.

2. $\frac{\pi}{2}-1$.

3. 略.

4. 略.

5. $\frac{256}{3}\pi$.

6. $\left(\frac{1}{a^2}+\frac{1}{b^2}\right)\frac{\pi R^4}{4}$.

7. $\frac{44}{15}\pi a^5$.

8. $\frac{4\pi R^3}{3a}$.

习题 11-1

1. (1) $2\pi a^7$. (2) $2\pi^2 a^3(1+2\pi^2)$. (3) $\frac{1}{12}(5\sqrt{5}-1)$. (4) $\sqrt{5}\ln 2$.

(5) $\frac{ab(a^2+ab+b^2)}{3(a+b)}$. (6) 24. (7) $\frac{7}{2}\sqrt{21}$.

2. 8.

习题 11-2

1. (1) $\frac{4}{3}ab^2$. (2) ① $-\frac{4}{3}a^3$. ② 0. (3) -8. (4) $-\frac{4}{3}$. (5) 13.

(6) $\frac{\sqrt{2}}{16}\pi$,提示:将曲线表示为参数方程 $x=\cos t, y=z=\frac{\sqrt{2}}{2}\sin t$.

2. $-|F|R$.

习题 11-3

1. (1) $\frac{1}{2}\pi a^4$. (2) -8. (3) 0. (4) $\frac{1}{2}\pi^2-\frac{1}{3}\pi^3$.

2. (1) 8. (2) $e^2\cos 1-1$.

3. 62.

习题 11-4

1. (1) $-\frac{27}{4}$. (2) $\frac{3-\sqrt{3}}{2}+(\sqrt{3}-1)\ln 2$. (3) πR^3. (4) $\frac{32}{9}\sqrt{2}$.

2. $\frac{\sqrt{2}}{2}\pi h^4$.

习题 11-5

1. $\iint\limits_{\Sigma}\left(\frac{3}{5}P+\frac{2}{5}Q+\frac{2\sqrt{3}}{5}R\right)\mathrm{d}S$.

2. (1) $\frac{4}{15}\pi a^5$. (2) $3a^3$. (3) $\frac{3}{2}\pi$. (4) -8π.

习题 11-6

1. (1) $-\frac{12\pi}{5}R$. (2) $\frac{1}{8}$. (3) $\frac{1}{2}\pi$.

2. $\frac{\pi}{2}$.

3. (1) $2(x+y+z)$. (2) $e^y+ze^{-y}+\frac{y}{z}$.

习题 11-7

1. (1) $-\sqrt{3}\pi a^2$. (2) -1.

2. (1) $2\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k}$. (2) $\boldsymbol{i}+\boldsymbol{j}$.

3. 12π.

复习题十一

一、选择题

1—5：BDBAC.　6—10：BDDDC.　11. D.　12. A.

二、综合练习 A

1. $\ln\frac{\sqrt{5}+3}{2}$.

2. 16.

3. 证明略，$u(x,y)=x^3-x^2y+xy^2-y^3+C$.

4. $-\pi$.

5. $\frac{3}{2}\pi a^4$.

6. $\frac{1}{3}h^2R^3$.

7. $3+\sin 1$.

8. $\frac{1}{2}\pi R^4$.

三、综合练习 B

1. -4π.

2. (1) 0.　(2) πR^3.

3. $\sin 1+\mathrm{e}-1$.

4. $\frac{3}{8}\pi a^2$.

5. $\frac{2\pi a^4}{\sqrt{1+a^2}}$.

6. 略.

习题 12-1

1. (1) $u_n=(-1)^{n-1}\frac{1}{2n-1}$ $(n=1,2,3,\cdots)$.　(2) $u_n=(-1)^{n-1}\frac{n^2}{(n+1)^2}$ $(n=1,2,3,\cdots)$.

2. $u_1=s_1=1, u_2=s_2-s_1=\frac{1}{3}, u_n=\begin{cases}1, & n=1,\\ \frac{2}{n(n+1)}, & n\geqslant 2.\end{cases}$

3. (1) 发散.　(2) 收敛.　(3) 收敛.

4. (1) 收敛.　(2) 发散.　(3) 发散.　(4) 发散.

习题 12-2

1. (1) 发散.　(2) 发散.　(3) 收敛.　(4) $a>1$ 时收敛，$0<a\leqslant 1$ 时发散.　(5) 发散.　(6) 发散.　(7) 发散.　(8) 收敛.

2. (1) 发散.　(2) 收敛.　(3) 收敛.　(4) 发散.　(5) 收敛.　(6) 发散.　(7) 收敛.　(8) 收敛.　(9) 收敛.　(10) $0<q<1$ 时，级数收敛；$q>1$ 时，级数发散；$q=1$ 时，当 $r>1$ 时，级数收敛，$r\leqslant 1$ 时，级数发散.

3.(1) 条件收敛. (2) 条件收敛. 提示:讨论 $n-\sqrt{n}$ 的单调性可考虑函数 $f(x)=x-\sqrt{x}$ 的单调性. (3) 发散. (4) 条件收敛. (5) 条件收敛. (6) 绝对收敛. (7) 绝对收敛. (8) 条件收敛.

习题 12-3

1.(1) $R=1,(-1,1)$. (2) $R=+\infty,(-\infty,+\infty)$. (3) $R=1,(-1,1)$. (4) $R=1,[-1,1)$. (5) $R=1,[-1,1]$. (6) $R=\frac{1}{3},\left[-\frac{1}{3},\frac{1}{3}\right]$. (7) $R=1,[-1,1]$. (8) $R=1,(0,2]$. (9) $R=\frac{\sqrt{2}}{2},\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$. (10) $R=1,\left[-1,1\right)$. (11) $R=\frac{1}{5},\left(-\frac{1}{5},\frac{1}{5}\right)$. 提示:把级数看作两个幂级数的和,收敛域取其公共部分. (12) $\left(\frac{1}{10},10\right)$.

2.(1) $\frac{1}{(1-x)^2}(-1<x<1)$.

(2) $-x+\frac{1}{2}\arctan x-\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| \quad (-1<x<1)$.

习题 12-4

1.(1) $\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n}x^n,(-2,2)$.

(2) $x+\sum_{n=2}^{\infty}\frac{(-1)^n x^n}{n(n-1)},(-1,1)$.

(3) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{n!},(-\infty,\infty)$.

(4) $\ln 3+\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n3^n}x^n,(-3,3]$.

(5) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2^{2n-1}}{(2n)!}x^{2n},(-\infty,+\infty)$.

(6) $\sum_{n=1}^{\infty}\frac{nx^{n-1}}{(n+1)!},(-\infty,\infty)$.

2. $\ln 2+\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-2)^n}{n\cdot 2^n},(0,4]$.

3. $\frac{1}{3}\sum_{n=0}^{\infty}(-1)^n\frac{(x-3)^n}{3^n},(0,6)$.

4. $\sum_{n=0}^{\infty}\frac{1}{3}\left[1+\frac{(-1)^n}{2^{n+1}}\right]x^{n+1}, \quad x\in(-1,1)$.

5. $\frac{1}{2}\sum_{n=0}^{\infty}\left[(-1)^n-\frac{(-1)^n}{3^{n+1}}\right]x^n=\sum_{n=0}^{\infty}\frac{(-1)^n}{2}\frac{3^{n+1}-1}{3^{n+1}}x^n,x\in(-1,1)$.

习题 12-5

1. 略.

2. 1.098 6.

3. 0.199 6.

4. 0.946 1.

习题 12-6

1.(1) $f(x)=-\frac{\pi}{4}+\left(\frac{2}{\pi}\cos x+\sin x\right)-\frac{1}{2}\sin 2x+\frac{2}{3^2\pi}\cos^3 x+\frac{1}{3}\sin 3x-\frac{1}{4}\sin 2x+$

$\left(\frac{2}{5^2\pi}\cos 5x+\frac{1}{5}\cos 5x\right)-\frac{1}{6}\sin 6x+\cdots(-\infty<x<+\infty,x\neq+\pi,+3\pi,\cdots)$.

(2) $f(x)=\frac{1}{2}+\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2k-1)x}{2k-1}\ (-\infty<x<+\infty,x\neq 0,\pm\pi,\pm 2\pi,\cdots)$.

(3) $f(x)=-\frac{\pi}{4}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2}+\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx\ (-\infty<x<+\infty,x\neq,\pm\pi,\pm 3\pi,\cdots)$.

(4) $f(x)=\frac{4}{\pi}\left[\sin x+\frac{1}{3}\sin 3x+\cdots+\frac{1}{2k-1}\sin(2k-1)x+\cdots\right]\ (-\infty<x<+\infty;\ x\neq k\pi,k=0,\pm 1,\pm 2,\cdots)$.

2. $f(x)=\frac{4}{\pi}\left[\sin x+\frac{1}{3}\sin 3x+\cdots+\frac{1}{2k-1}\sin(2k-1)x+\cdots\right](0\leqslant x<\pi)$.

3. $f(x)=\sum_{n=1}^{\infty}\frac{1}{n}\sin nx\ (0<x\leqslant\pi)$.

4. $f(x)=\pi+1-\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos(2n-1)x\quad(0\leqslant x\leqslant\pi)$.

5. $\frac{\pi^3}{8}$,0.

6. 略

习题 12-7

1. $f(x)=\frac{11}{12}+\frac{1}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos 2n\pi x\ (-\infty<x<+\infty)$.

2. $\frac{4}{3}+\frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\frac{n\pi}{2}x+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi}{2}x$.

3. $f(x)=\frac{8}{\pi}\left[\frac{1}{3}\sin\pi x+\frac{2}{15}\sin 2\pi x+\frac{3}{35}\sin 3\pi x+\cdots\right](0<x\leqslant 1)$.

4. $f(x)=\frac{pa}{8}-\frac{pa}{\pi^2}\left[\cos\frac{2\pi x}{a}+\frac{1}{3^2}\cos\frac{6\pi x}{a}+\frac{1}{5^2}\cos\frac{10\pi x}{a}+\cdots\right](0\leqslant x\leqslant a)$.

5. 正弦级数：$f(x)=\frac{8}{\pi^2}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{(2n-1)^2}\sin\frac{(2n-1)\pi x}{2}\ (0\leqslant x\leqslant 2)$；

余弦级数：$f(x)=\frac{1}{2}-\frac{4}{\pi^2}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{(2n-1)^2}\sin(2n-1)\pi x\ (0\leqslant x\leqslant 2)$.

复习题十二

一、选择题

1—5：BCCDA.　6—10：CBBBB.　11—15：BACAC.　16. D.　17. B.

二、综合练习 A

1. 提示：利用两级数部分和的关系证明.

2. 提示：利用级数收敛的必要条件.

3. (1) 发散.　(2) 条件收敛.　(3) 发散.

4. 不能. 所给例子是发散的级数.

5. $(-1,1)$. 提示：利用比值法可求得 $R=1$.

6. 略.

7. $1-\frac{1}{3\cdot 3!}+\frac{1}{5\cdot 5!}-\frac{1}{7\cdot 7!}+\cdots$.

三、综合练习 B

1. 略.

2. (1) 发散.　(2) 收敛.

3. $(-e,e)$.

4. $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+2}-\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}x^{2n+2}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)(2n+2)}x^{2n+2}(-1\leqslant x\leqslant 1)$.

5. $-1<x-1<1, s(x)=\frac{1}{(2-x)^2}$.

6. $-1<x<1, x\neq 0, s(x)=\frac{x}{1-x}+\frac{1}{x}\ln(1-x)$. $s(0)=0$.

7. 绝对收敛.　8. $0<x_n<\frac{1}{n}$. 证明略.

9. $f(x)=\frac{5}{2}-\frac{4}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\cos(2k-1)\pi x\ (-1\leqslant x\leqslant 1)$, $\frac{\pi^2}{6}$.